

Chapter 6

Risk, Return, and the Capital Asset Pricing Model

ANSWERS TO END-OF-CHAPTER QUESTIONS

- 6-1
- a. Stand-alone risk is only a part of total risk and pertains to the risk an investor takes by holding only one asset. Risk is the chance that some unfavorable event will occur. For instance, the risk of an asset is essentially the chance that the asset's cash flows will be unfavorable or less than expected. A probability distribution is a listing, chart or graph of all possible outcomes, such as expected rates of return, with a probability assigned to each outcome. When in graph form, the tighter the probability distribution, the less uncertain the outcome.
 - b. The expected rate of return ($\hat{r}$) is the expected value of a probability distribution of expected returns.
 - c. A continuous probability distribution contains an infinite number of outcomes and is graphed from $-\infty$ and $+\infty$.
 - d. The standard deviation (σ) is a statistical measure of the variability of a set of observations. The variance (σ^2) of the probability distribution is the sum of the squared deviations about the expected value adjusted for deviation. The coefficient of variation (CV) is equal to the standard deviation divided by the expected return; it is a standardized risk measure which allows comparisons between investments having different expected returns and standard deviations.
 - e. A risk averse investor dislikes risk and requires a higher rate of return as an inducement to buy riskier securities. A realized return is the actual return an investor receives on their investment. It can be quite different than their expected return.
 - f. A risk premium is the difference between the rate of return on a risk-free asset and the expected return on Stock i which has higher risk. The market risk premium is the difference between the expected return on the market and the risk-free rate.
 - g. CAPM is a model based upon the proposition that any stock's required rate of return is equal to the risk free rate of return plus a risk premium reflecting only the risk remaining after diversification.

- h. The expected return on a portfolio, $\hat{r}_p$, is simply the weighted-average expected return of the individual stocks in the portfolio, with the weights being the fraction of total portfolio value invested in each stock. The market portfolio is a portfolio consisting of all stocks.
 - i. Correlation is the tendency of two variables to move together. A correlation coefficient (ρ) of +1.0 means that the two variables move up and down in perfect synchronization, while a coefficient of -1.0 means the variables always move in opposite directions. A correlation coefficient of zero suggests that the two variables are not related to one another; that is, they are independent.
 - j. Market risk is that part of a security's total risk that cannot be eliminated by diversification. It is measured by the beta coefficient. Diversifiable risk is also known as company specific risk, that part of a security's total risk associated with random events not affecting the market as a whole. This risk can be eliminated by proper diversification. The relevant risk of a stock is its contribution to the riskiness of a well-diversified portfolio.
 - k. The beta coefficient is a measure of a stock's market risk, or the extent to which the returns on a given stock move with the stock market. The average stock's beta would move on average with the market so it would have a beta of 1.0.
 - l. The security market line (SML) represents in a graphical form, the relationship between the risk of an asset as measured by its beta and the required rates of return for individual securities. The SML equation is essentially the CAPM, $r_i = r_{RF} + b_i(r_M - r_{RF})$.
 - m. The slope of the SML equation is $(r_M - r_{RF})$, the market risk premium. The slope of the SML reflects the degree of risk aversion in the economy. The greater the average investors aversion to risk, then the steeper the slope, the higher the risk premium for all stocks, and the higher the required return.
- 6-2
- a. The probability distribution for complete certainty is a vertical line.
 - b. The probability distribution for total uncertainty is the X axis from $-\infty$ to $+\infty$.
- 6-3
- Security A is less risky if held in a diversified portfolio because of its lower beta and negative correlation with other stocks. In a single-asset portfolio, Security A would be more risky because $\sigma_A > \sigma_B$ and $CV_A > CV_B$.

- 6-4
- No, it is not riskless. The portfolio would be free of default risk and liquidity risk, but inflation could erode the portfolio's purchasing power. If the actual inflation rate is greater than that expected, interest rates in general will rise to incorporate a larger inflation premium (IP) and the value of the portfolio would decline.
 - No, you would be subject to reinvestment rate risk. You might expect to "roll over" the Treasury bills at a constant (or even increasing) rate of interest, but if interest rates fall, your investment income will decrease.
 - A U.S. government-backed bond that provided interest with constant purchasing power (that is, an indexed bond) would be close to riskless.
- 6-5 The risk premium on a high beta stock would increase more.

$$RP_j = \text{Risk Premium for Stock } j = (r_M - r_{RF})b_j.$$

If risk aversion increases, the slope of the SML will increase, and so will the market risk premium ($r_M - r_{RF}$). The product $(r_M - r_{RF})b_j$ is the risk premium of the j th stock. If b_j is low (say, 0.5), then the product will be small; RP_j will increase by only half the increase in RP_M . However, if b_j is large (say, 2.0), then its risk premium will rise by twice the increase in RP_M .

- 6-6 According to the Security Market Line (SML) equation, an increase in beta will increase a company's expected return by an amount equal to the market risk premium times the change in beta. For example, assume that the risk-free rate is 6 percent, and the market risk premium is 5 percent. If the company's beta doubles from 0.8 to 1.6 its expected return increases from 10 percent to 14 percent. Therefore, in general, a company's expected return will not double when its beta doubles.
- 6-7 Yes, if the portfolio's beta is equal to zero. In practice, however, it may be impossible to find individual stocks that have a nonpositive beta. In this case it would also be impossible to have a stock portfolio with a zero beta. Even if such a portfolio could be constructed, investors would probably be better off just purchasing Treasury bills, or other zero beta investments.

SOLUTIONS TO END-OF-CHAPTER PROBLEMS

6-1	Investment	Beta
	\$35,000	0.8
	<u>40,000</u>	1.4
Total	<u>\$75,000</u>	

$$(\$35,000/\$75,000)(0.8) + (\$40,000/\$75,000)(1.4) = 1.12.$$

6-2 $r_{RF} = 6\%$; $r_M = 13\%$; $b = 0.7$; $r_s = ?$

$$\begin{aligned} r_s &= r_{RF} + (r_M - r_{RF})b \\ &= 6\% + (13\% - 6\%)0.7 \\ &= 10.9\%. \end{aligned}$$

6-3 $r_{RF} = 5\%$; $RP_M = 6\%$; $r_M = ?$

$$r_M = 5\% + (6\%)1 = 11\%.$$

r_s when $b = 1.2 = ?$

$$r_s = 5\% + 6\%(1.2) = 12.2\%.$$

6-4 $\hat{r} = (0.1)(-50\%) + (0.2)(-5\%) + (0.4)(16\%) + (0.2)(25\%) + (0.1)(60\%)$
 $= 11.40\%.$

$$\begin{aligned} \sigma^2 &= (-50\% - 11.40\%)^2(0.1) + (-5\% - 11.40\%)^2(0.2) + (16\% - 11.40\%)^2(0.4) \\ &\quad + (25\% - 11.40\%)^2(0.2) + (60\% - 11.40\%)^2(0.1) \\ \sigma^2 &= 712.44; \sigma = 26.69\%. \end{aligned}$$

$$CV = \frac{26.69\%}{11.40\%} = 2.34.$$

6-5 a. $\hat{r}_m = (0.3)(15\%) + (0.4)(9\%) + (0.3)(18\%) = 13.5\%.$

$$\hat{r}_j = (0.3)(20\%) + (0.4)(5\%) + (0.3)(12\%) = 11.6\%.$$

b. $\sigma_M = [(0.3)(15\% - 13.5\%)^2 + (0.4)(9\% - 13.5\%)^2 + (0.3)(18\% - 13.5\%)^2]^{1/2}$
 $= \sqrt{14.85\%} = 3.85\%.$

$$\sigma_J = [(0.3)(20\% - 11.6\%)^2 + (0.4)(5\% - 11.6\%)^2 + (0.3)(12\% - 11.6\%)^2]^{1/2}$$

$$= \sqrt{38.64\%} = 6.22\%.$$

c. $CV_M = \frac{3.85\%}{13.5\%} = 0.29.$

$$CV_J = \frac{6.22\%}{11.6\%} = 0.54.$$

6-6 a. $r_A = r_{RF} + (r_M - r_{RF})b_A$
 $12\% = 5\% + (10\% - 5\%)b_A$
 $12\% = 5\% + 5\%(b_A)$
 $7\% = 5\%(b_A)$
 $1.4 = b_A.$

b. $r_A = 5\% + 5\%(b_A)$
 $r_A = 5\% + 5\%(2)$
 $r_A = 15\%.$

6-7 a. $r_i = r_{RF} + (r_M - r_{RF})b_i = 9\% + (14\% - 9\%)1.3 = 15.5\%$.

b. 1. r_{RF} increases to 10%:

r_M increases by 1 percentage point, from 14% to 15%.

$$r_i = r_{RF} + (r_M - r_{RF})b_i = 10\% + (15\% - 10\%)1.3 = 16.5\%.$$

2. r_{RF} decreases to 8%:

r_M decreases by 1%, from 14% to 13%.

$$r_i = r_{RF} + (r_M - r_{RF})b_i = 8\% + (13\% - 8\%)1.3 = 14.5\%.$$

c. 1. r_M increases to 16%:

$$r_i = r_{RF} + (r_M - r_{RF})b_i = 9\% + (16\% - 9\%)1.3 = 18.1\%.$$

2. r_M decreases to 13%:

$$r_i = r_{RF} + (r_M - r_{RF})b_i = 9\% + (13\% - 9\%)1.3 = 14.2\%.$$

6-8 Old portfolio beta = $\frac{\$142,500}{\$150,000}(b) + \frac{\$7,500}{\$150,000}(1.00)$
 $1.12 = 0.95b + 0.05$
 $1.07 = 0.95b$
 $1.13 = b.$

$$\text{New portfolio beta} = 0.95(1.13) + 0.05(1.75) = 1.16.$$

Alternative Solutions:

1. Old portfolio beta = $1.12 = (0.05)b_1 + (0.05)b_2 + \dots + (0.05)b_{20}$

$$1.12 = (\Sigma b_i)(0.05)$$

$$\Sigma b_i = 1.12/0.05 = 22.4.$$

$$\text{New portfolio beta} = (22.4 - 1.0 + 1.75)(0.05) = 1.1575 = 1.16.$$

2. Σb_i excluding the stock with the beta equal to 1.0 is $22.4 - 1.0 = 21.4$, so the beta of the portfolio excluding this stock is $b = 21.4/19 = 1.1263$. The beta of the new portfolio is:

$$1.1263(0.95) + 1.75(0.05) = 1.1575 = 1.16.$$

6-9 Portfolio beta =
$$\begin{aligned} & \frac{\$400,000}{\$4,000,000}(1.50) + \frac{\$600,000}{\$4,000,000}(-0.50) \\ & + \frac{\$1,000,000}{\$4,000,000}(1.25) + \frac{\$2,000,000}{\$4,000,000}(0.75) \\ & = 0.1(1.5) + (0.15)(-0.50) + (0.25)(1.25) + (0.5)(0.75) \\ & = 0.15 - 0.075 + 0.3125 + 0.375 = 0.7625. \end{aligned}$$

$$r_p = r_{RF} + (r_M - r_{RF})(b_p) = 6\% + (14\% - 6\%)(0.7625) = 12.1\%.$$

Alternative solution: First compute the return for each stock using the CAPM equation $[r_{RF} + (r_M - r_{RF})b]$, and then compute the weighted average of these returns.

$$r_{RF} = 6\% \text{ and } r_M - r_{RF} = 8\%.$$

Stock	Investment	Beta	$r = r_{RF} + (r_M - r_{RF})b$	Weight
A	\$ 400,000	1.50	18%	0.10
B	600,000	(0.50)	2	0.15
C	1,000,000	1.25	16	0.25
D	2,000,000	0.75	12	0.50
Total	<u>\$4,000,000</u>			<u>1.00</u>

$$r_p = 18\%(0.10) + 2\%(0.15) + 16\%(0.25) + 12\%(0.50) = 12.1\%.$$

- 6-10 First, calculate the beta of what remains after selling the stock:

$$\begin{aligned} b_p &= 1.1 = (\$100,000/\$2,000,000)0.9 + (\$1,900,000/\$2,000,000)b_R \\ 1.1 &= 0.045 + (0.95)b_R \\ b_R &= 1.1105. \end{aligned}$$

$$b_N = (0.95)1.1105 + (0.05)1.4 = 1.125.$$

6-11 We know that $b_R = 1.50$, $b_S = 0.75$, $r_M = 13\%$, $r_{RF} = 7\%$.

$$r_i = r_{RF} + (r_M - r_{RF})b_i = 7\% + (13\% - 7\%)b_i.$$

$$r_R = 7\% + 6\%(1.50) = 16.0\%$$

$$r_S = 7\% + 6\%(0.75) = \underline{11.5\%}$$

6-12 The answers to a, b, c, and d are given below:

	$\bar{r}_A$	$\bar{r}_B$	Portfolio
2006	(18.00%)	(14.50%)	(16.25%)
2007	33.00	21.80	27.40
2008	15.00	30.50	22.75
2009	(0.50)	(7.60)	(4.05)
2010	27.00	26.30	26.65
Mean	11.30	11.30	11.30
Std Dev	20.79	20.78	20.13
CV	1.84	1.84	1.78

e. A risk-averse investor would choose the portfolio over either Stock A or Stock B alone, since the portfolio offers the same expected return but with less risk. This result occurs because returns on A and B are not perfectly positively correlated ($\rho_{AB} = 0.88$).

6-13 a. $b_X = 1.3471$; $b_Y = 0.6508$. These can be calculated with a spreadsheet.

b. $r_X = 6\% + (5\%)1.3471 = 12.7355\%$.

$r_Y = 6\% + (5\%)0.6508 = 9.2540\%$.

c. $b_p = 0.8(1.3471) + 0.2(0.6508) = 1.2078$.

$r_p = 6\% + (5\%)1.2078 = 12.04\%$.

Alternatively,

$r_p = 0.8(12.7355\%) + 0.2(9.254\%) = 12.04\%$.

d. Stock X is undervalued, because its expected return exceeds its required rate of return.